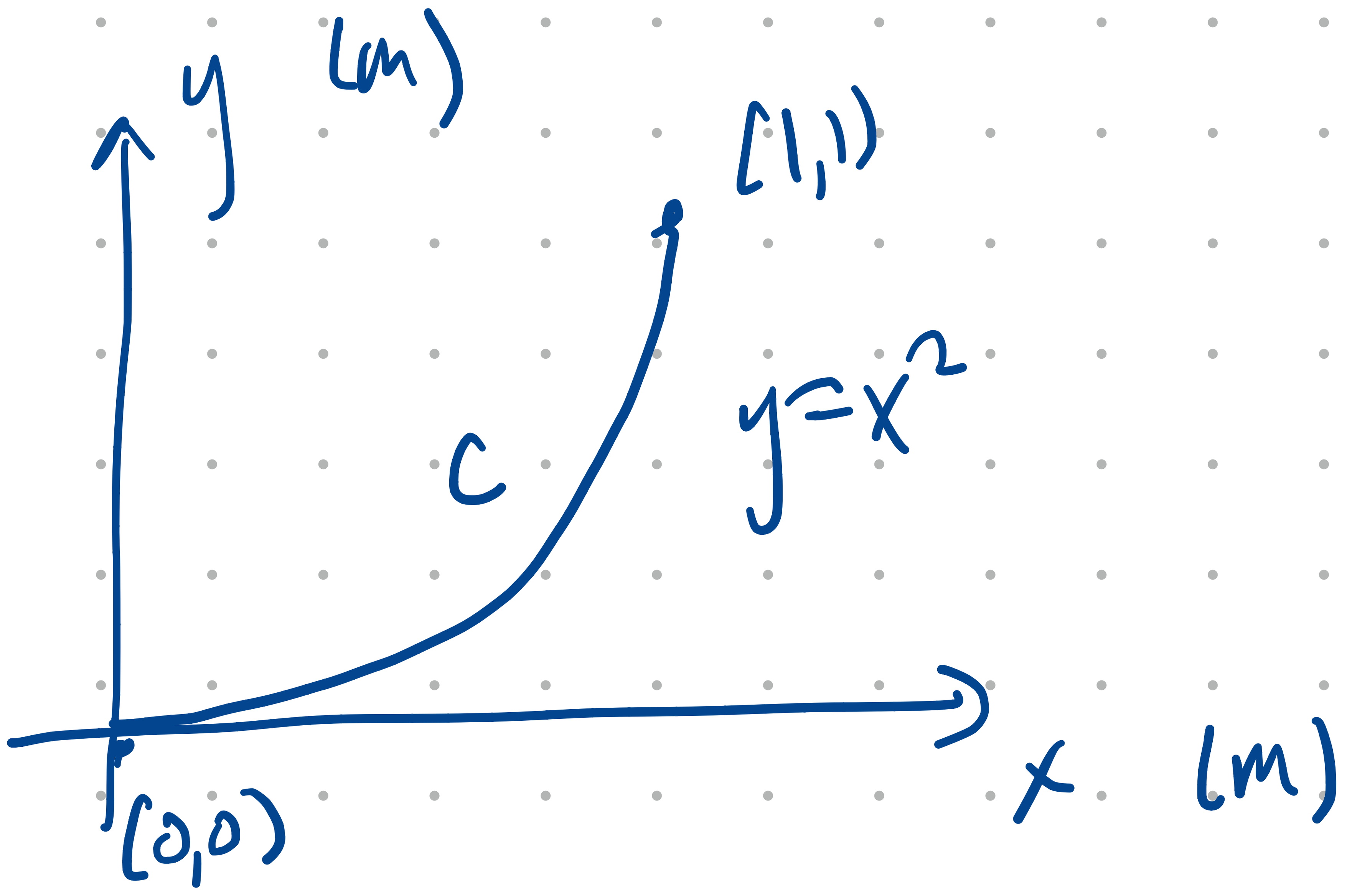
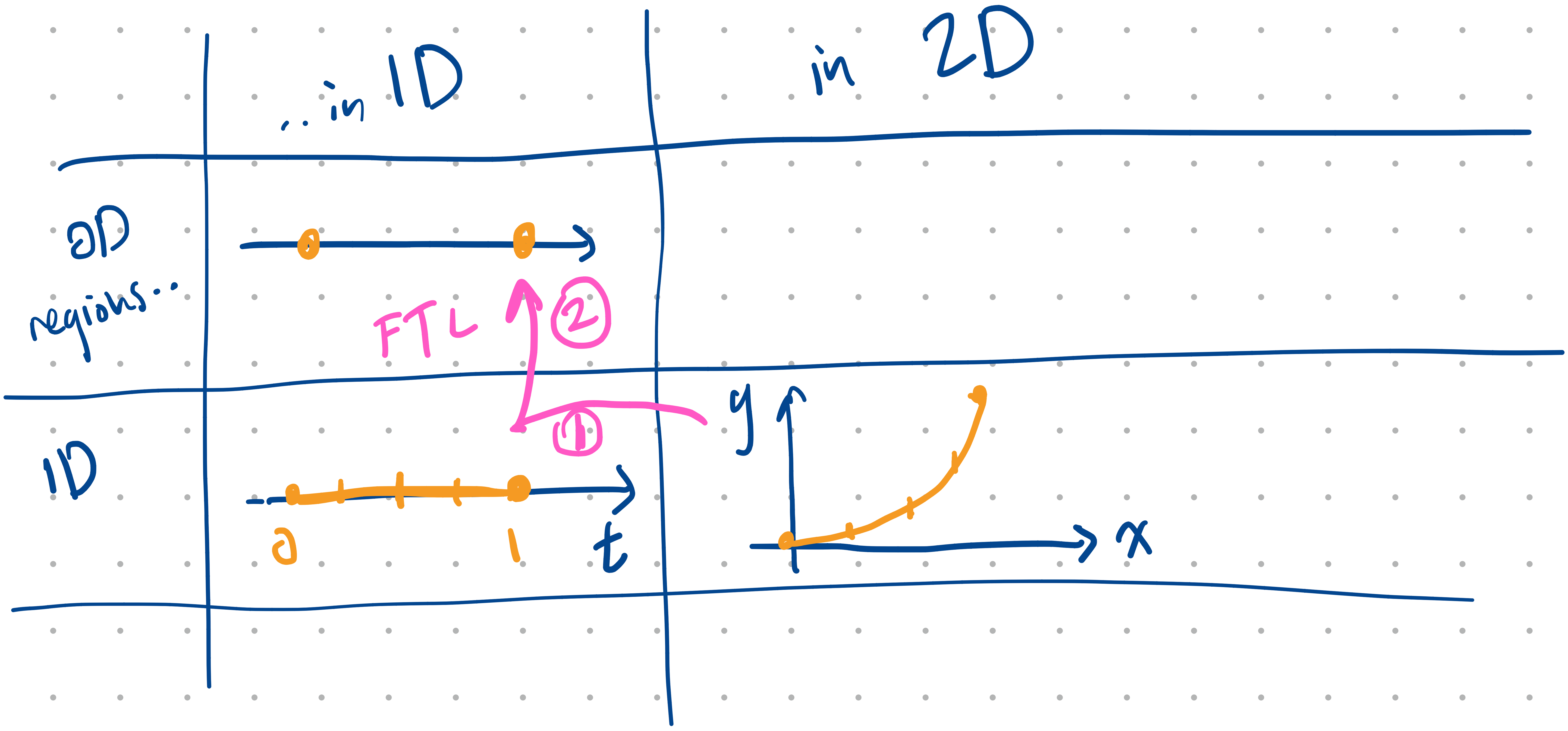
Ex) What is the mass of a cable in the shape



with density $\sigma(x,y) = \sqrt{y}$ $\left(\text{kg}/\text{m}\right)$?

$$\text{Mass} = \int_C dm = \int_C \sigma(x,y)\, ds = \int_C \sqrt{y}\, ds$$



① Parametrize C

$\vec{r} = \langle x, y \rangle$ $\quad x = t$ natural choice of parameter

$\vec{r}(t) = \langle t, t^2 \rangle$. $\quad 0 \le t \le 1$

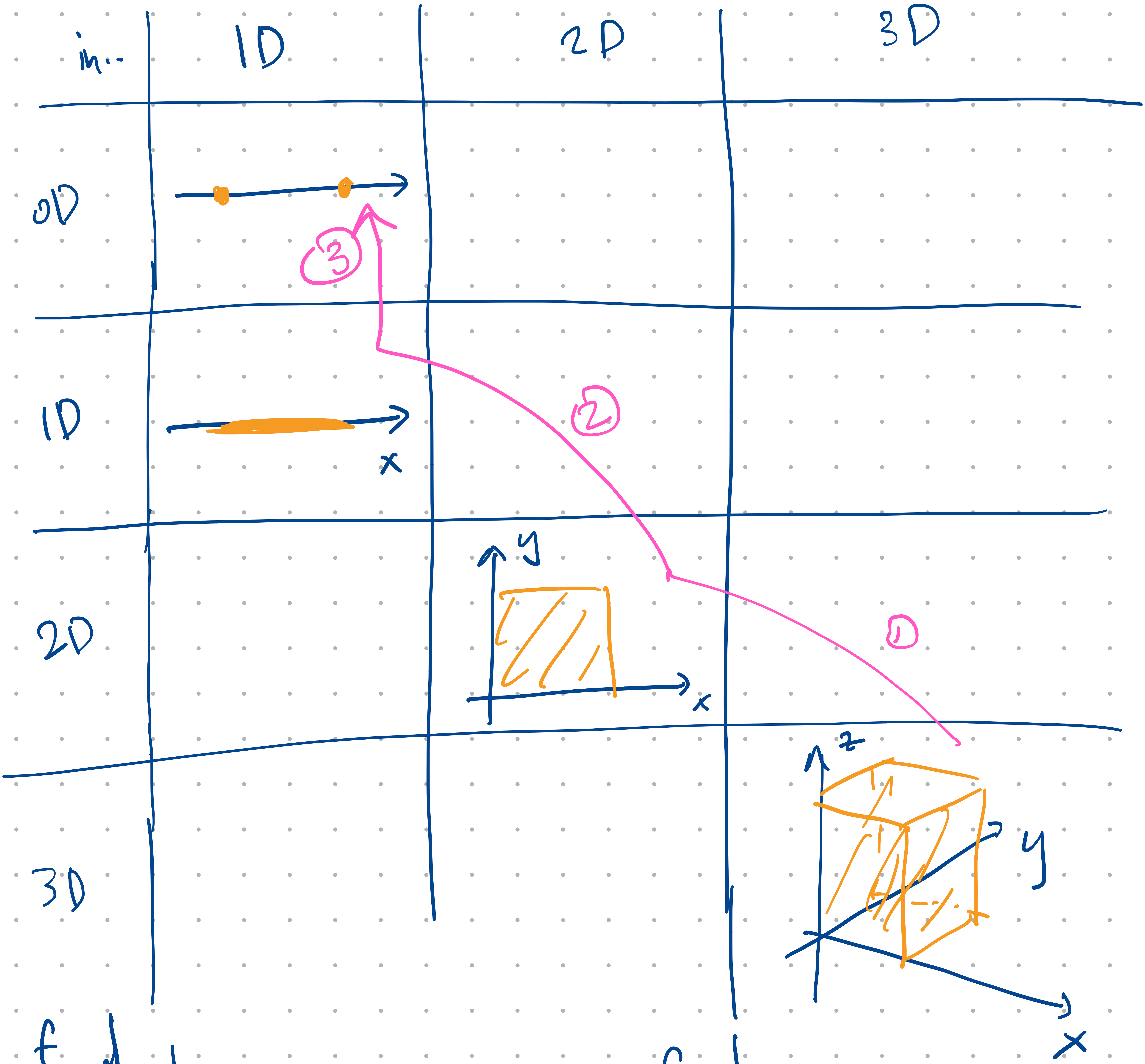
Recall: $ds = |\vec{r}\,'(t)|\,dt$, since $\frac{ds}{dt} = |\vec{r}\,'(t)|$

"change in distance over time = speed = magnitude of velocity"

$\vec{r}\,'(t) = \langle 1, 2t \rangle$ $\quad$ so $\quad |\vec{r}\,'(t)| = \sqrt{1^2 + (2t)^2}$

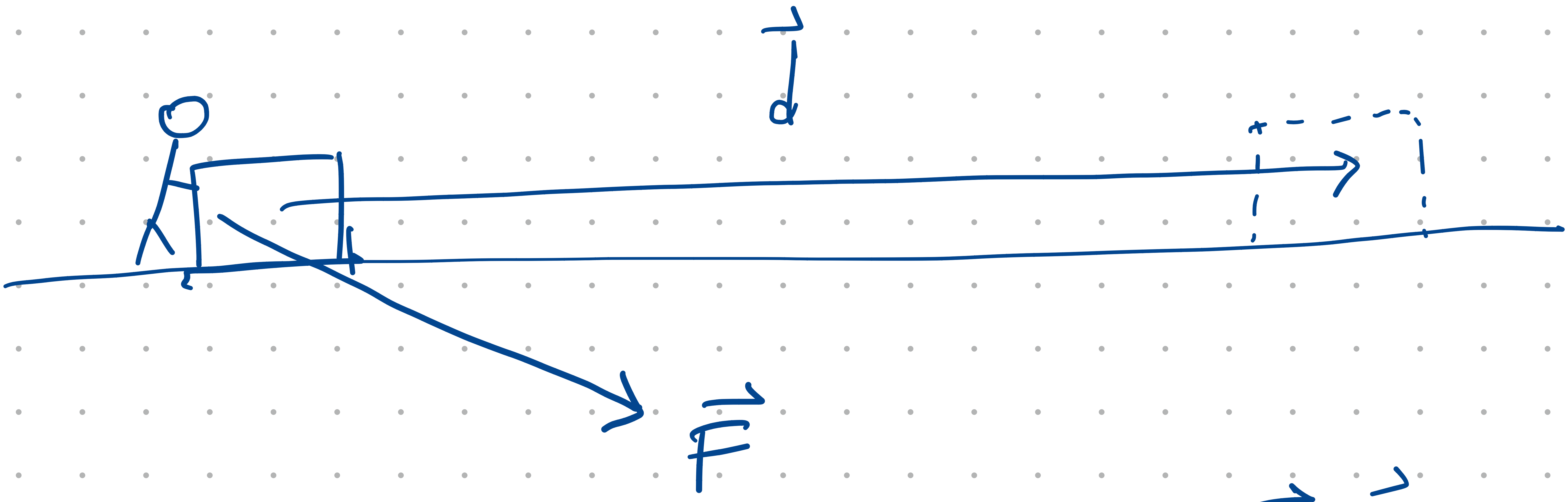
$$\int_C \sqrt{y}\, ds = \int_0^1 \sqrt{t^2}\,\sqrt{1 + 4t^2}\, dt$$

$$= \int_0^1 t\sqrt{1+4t^2}\, dt \overset{②}{=} \frac{1}{12}(1+4t^2)^{3/2} \Big|_{t=0}^{1} = \ldots$$



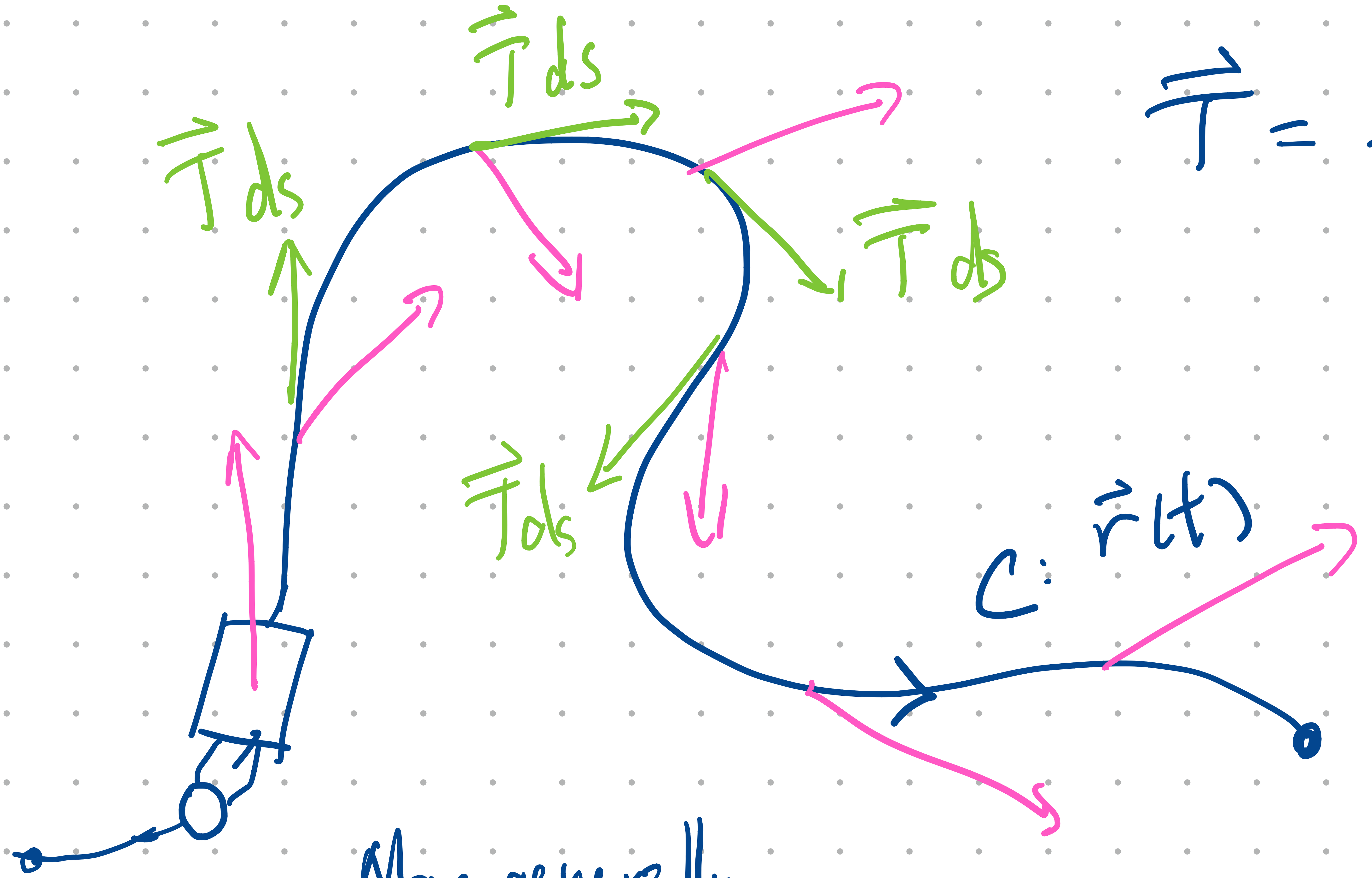
$$\int_e^f \int_c^d \int_a^b \cdots \, dz\, dy\, dx \overset{1}{=} \int_e^f \int_c^d \cdots \, dy\, dx$$

$$\overset{2}{=} \int_e^f \cdots \, dx \overset{3}{=} \left(\cdots \right) \Big|_{x=e}^{f}$$



<u>Defn</u>. The work W done by this force is $\vec{F}\cdot\vec{d}$.

This assumes $\vec{F}$ is <u>constant</u>.



$$\vec{T} = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}$$

More generally...

$$\int_C \vec{F} \cdot \vec{T}\, ds = \int_a^b \vec{F} \cdot \frac{\vec{r}'(t)}{\cancel{|\vec{r}'(t)|}} \cancel{|\vec{r}'(t)|}\, dt$$

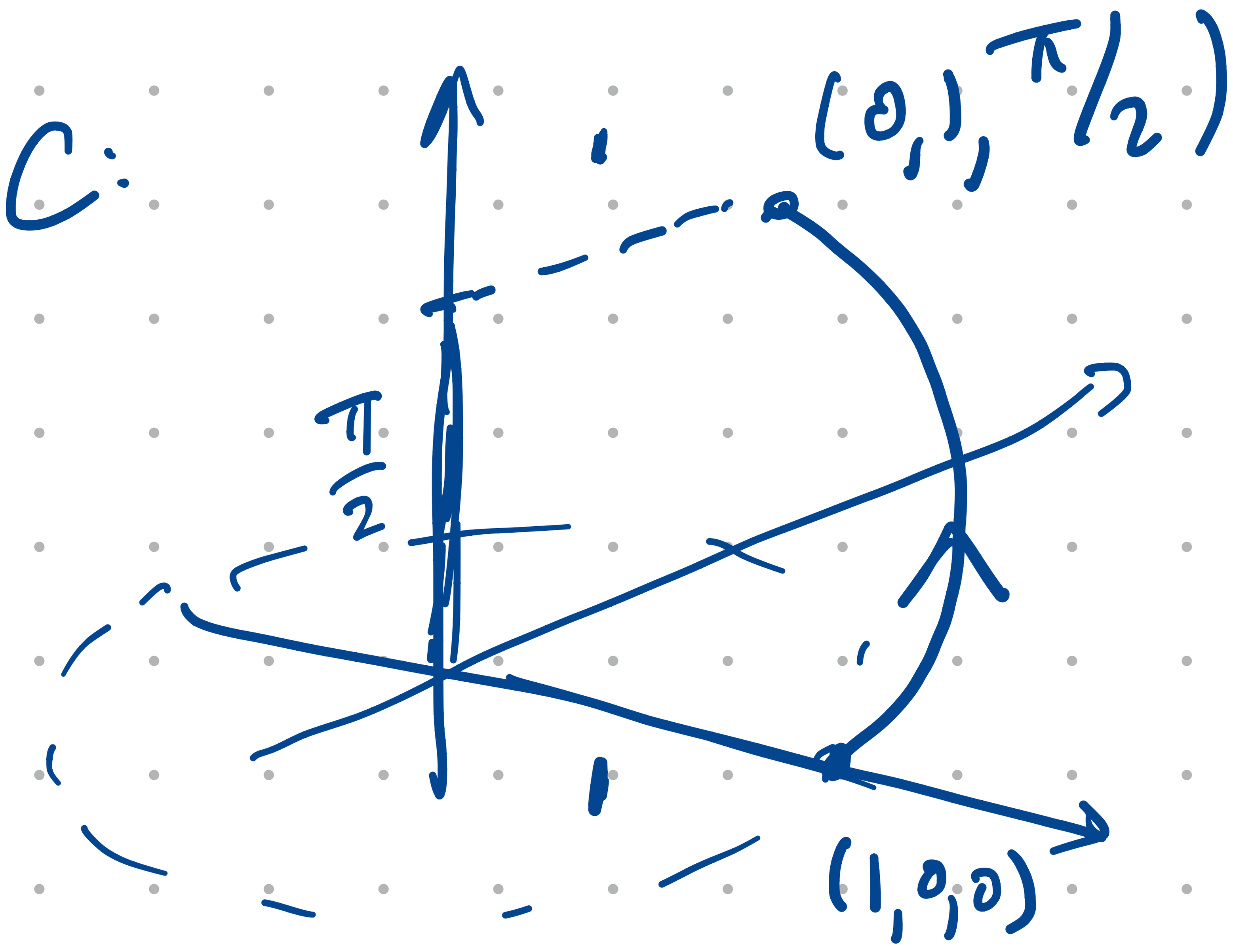
may now be varying

$$= \int_a^b \vec{F} \cdot \vec{r}'(t)\, dt$$

$$\boxed{\int_C \vec{F} \cdot d\vec{r}}$$

Ex) $\displaystyle\int_C \vec{F} \cdot d\vec{r}$

$\vec{F} = \langle x, z, y \rangle$.

C: 

$\vec{r}(t) = \langle \cos t, \sin t, t \rangle$

$0 \le t \le \frac{\pi}{2}$

$$= \int_0^{\pi/2} \quad dt$$

⚠ If the orientation of C were reversed, $\vec{T}$ would be replaced by $-\vec{T}$ and the integral would be different by a sign.

Let $\vec{F}(t)$, $\vec{G}(t)$ be vec. valued fns. of t.

$$(\vec{F}\cdot\vec{G})' = \vec{F}'\cdot\vec{G} + \vec{F}\cdot\vec{G}' \quad \text{(check!)}$$

Ex)

$$\vec{F} = m\vec{a}$$

$$W \underset{\text{by defn}}{=} \int_C \vec{F}\cdot d\vec{r} = \int_C m\vec{a}\cdot d\vec{r}$$

$$= \int_{t_0}^{t_1} m\vec{r}''(t)\cdot\vec{r}'(t)\,dt$$

$$= \left(\frac{1}{2}m\vec{r}'\cdot\vec{r}'\right)\Bigg|_{t=t_0}^{t_1} = \underbrace{\left(\frac{1}{2}mv^2\right)}_{\text{Kinetic Energy}}\Bigg|_{t=t_0}^{t_1} = \Delta KE$$